# Mathématiques <br> <br> gANE <br> <br> gANE <br> <br> Frontières 

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The Scottish Mathematical Council

Partial solutions and attempts can earn marks.
Neat and careful work is important.
Submit only one team answer sheet for each question

## Q. 1 With Two Hands (7 points)

Give your answers in Spanish, German, Italian, or French using a minimum of 30 words.

Paolo a dans une main un nombre pair de pièces et dans l'autre un nombre impair de pièces. Il demande à Emma de deviner dans quelle main se trouve le nombre pair de pièces.
Alors Emma lui demande de multiplier le nombre de pièces de la main droite par deux et le nombre de pièces de la main gauche par trois.

Ensuite, elle lui demande de faire la somme des deux produits obtenus.

Expliquer comment, avec cette somme, Emma peut déterminer dans quelle main se trouve le nombre pair de pièces.

Paolo tiene un número par de monedas en una mano y un número impar en la otra. Le pregunta a Emma que adivine en qué mano se encuentra el número par de monedas.

Entonces Emma le pide que multiplique el número de monedas de la mano derecha por dos y el número de monedas de la mano izquierda por tres.

Luego, le pide que sume los dos productos obtenidos.

Explica cómo, con esta suma, Emma puede determinar en qué mano se encuentra el número par de.


Paolo che ha in una mano un numero pari di pedine e nell'altra un numero dispari domanda a Emma d'indovinare in quale mano si trovino le pedine in numero pari.

Emma chiede, quindi, al suo amico Paolo di moltiplicare per due il numero delle pedine contenute nella mano destra e per tre quelle contenute nella mano sinistra e, poi, di sommare i due prodotti ottenuti.

Spiegate come Emma, con questa somma, possa individuare quale mano

Paolo hat in der einen Hand eine gerade Anzahl und in der anderen Hand eine ungerade Anzahl von Münzen. Emma soll erraten, in welcher Hand sich die gerade Anzahl von Münzen befindet.

Emma bittet inn, die Anzahl der Münzen in der rechten Hand mit zwei und die Anzahl der Münzen in der linken Hand mit drei zu multiplizieren.

Danach soll Paolo die Produkte, die er erhalten hat, addieren.

Erklärt, wie Emma mit dieser Summe bestimmen kann, in welcher Hand sich die gerade Anzahl von Münzen befindet.

## Q. 2 Downright Square (5 points)

I am a four digit number.
I am the square of a number.
Also, the numbers formed by my first two digits and my last two digits are different non-zero squares.

## Who am I?

## Explain the process.



## Q. 3 The Triangles (7 points)

I play with three fair six-sided dice, as follows:
I successively roll the three dice and with the three numbers obtained I try to construct a triangle.

For example, with 4, 5 and 6 I can build a triangle whose sides are $4 \mathrm{~cm}, 5 \mathrm{~cm}$ and 6 cm , but with 1,2 and 6 I cannot.

I win if I can build a planar triangle.
I lose if I can't build a triangle.
What is the probability of winning if the first die gives the number 1?
What is the probability of winning, if the first die gives the number $4 ?$


## Q. 4 Two Speed (5 points)

To go shopping, Theo takes his electric scooter. On the outward journey, it moves at an average speed of $6 \mathrm{~km} / \mathrm{h}$. On the way back, for fear of being late, he takes the same path and rides at an average speed of $14 \mathrm{~km} / \mathrm{h}$.

The screen of his scooter shows him his average speed over the entire trip. He is surprised that it is not $10 \mathrm{~km} / \mathrm{h}$.

Determine its average speed over the entire trip. Explain your answer.


## Q. 5 Rising Stairs (7 points)

Laure and Mickaël are planning to participate in the Mont Niesen stair race in Switzerland. To succeed in the event, each athlete has a specific rhythm.

Laure climbs the steps two by two and counts her steps.

Mickaël climbs the steps three by three and counts his steps. Out of breath, Mickaël stops. Laure joins him on the same step. At this point, Laure counted 250 steps more than Mickaël.

How many steps have they already climbed? Justify your answer.


## Q. 6 Point of Error (5 points)



The cashier forgot to write the decimal point on the bank card payment terminal when François paid for petrol for his scooter.

A mistake that costs François $€ 1,826.55$ too much!
Obtain the amount that the cashier should have indicated on the payment terminal.

Explain your answer.

## Q. 7 At Stake (7 points)

To tie tomato plants in the vegetable garden, a garden center sells sets of three stakes.

They are in the shape of cylinders, 6 cm in diameter and of the same length.
They are sold in sets of three, tied together by two tight strings.
It takes 20 cm to tie a knot in the string.
Draw the cross-section of a set of three stakes at the position of a tied string.

What minimum length of string, to the nearest cm , should the garden center have per set of stakes?

Justify your answer.


## Q. 8 On Your Marks! (5 points)

Here is a simplified diagram which is not to scale - of an athletics stadium for the 200 m sprint.

Three riders start from points $A, B$ and $C$. The finish line is at the location of the flag.

Explain why the starting lines are staggered.

Calculate, to the nearest cm , the distances a and b
 between these starting lines.

## Q. 9 Double Meaning (7 points)

Find a four digit abcd number which, when multiplied by 4, gives dcba.

Explain your answer.


## Q. 10 Discovery (10 points)

In a geometry book, I found a construction that multiplies the area of a disk by 2 ; by 3 ; etc.

Construct the first three circles from a circle with center $O$ and diameter [AB] with $A B=8 \mathrm{~cm}$.

Prove that the area of the disc passing through $D$ is double that of the starting disc.

Prove that the area of the disc passing through $F$ is three times that of the starting disc.


How could you continue to obtain a method allowing you to multiply the area by 4 , by 5 ?

## Senior Classes Only

## Q. 11 A Necessary Break (5 points)

Élio takes a short 35-minute break to play his favorite video game.

When he begins his game, he notices that his digital clock displays four consecutive digits, but not necessarily displayed in order.

At the end of the game, he is surprised: "Here, the displayed time still has four consecutive digits, it's strange! "

## At what time did Elio start his break?



Justify your answer.

## Q. 12 This Is Silly (7 points)

The set of points located exactly 1 m from a cube with a side of 1 m traces a solid. Calculate the volume of this solid.


## Q. 13 Pay Attention to the Appearance

## (10 points)

The children of the municipal youth council proposed to the town hall of Strasbourg to build, for the winter holidays, on the square in front of the Place Kléber, an ephemeral skating rink in the shape of an oval, made up of arcs of circles of centers $C, D, E$ and $F$. The length $A B$ measures 30 m .

To present this project to technical services, you need two documents:

- a layout of the ice rink on a scale of $1 / 200$;
- a calculation of the length (to the nearest centimeter) of the necessary protective barrier all around the rink. This barrier is shown in bold in the drawing.


## Produce these two documents.



